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Weak Lensing, Shear and the Cosmic Virial Theorem in a Model with a Scale-Dependent Gravitational Coupling

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Abstract

It is argued that, in models where the gravitational coupling is scale-dependent, predictions concerning weak gravitational lensing and shear are essentially similar to the ones derived from General Relativity. This is consistent with recent negative results of observations of the MS1224, CL2218 and A1689 systems aimimg to infer from those methods the presence of dark matter. It is shown, however, that the situation is quite different when an analysis based on the Cosmic Virial Theorem is concerned.

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It has been recently reported that studies of the distortion of the surface brightness of background faint galaxies due to the gravitational shear of MS1224.7+2007 are consistent with an excess of mass of about a factor 3 in relation to the one inferred from the mass-to-light ratio analysis [1]. This excess has been attributed to the presence of dark matter. Distortion effects of this nature were also reported for the CL 0024 system [2] and similar studies were performed for the CL 1455 and CL 0016+16 systems [3]. More recent studies indicate, however, that discrepancies are much smaller and can be accounted for by internal dynamical effects other than the presence of dark matter. Furthermore, it is found that for systems such as A 2218 the mass estimated by the weak gravitational lensing method is consistent with the one inferred from the X-ray data [4], while in different studies for the A2218, A1689 and A2163 systems, discrepancies actually suggest that the inferred mass distribution is somewhat more compact than the one arising from studies of the X-ray profiles [5]. Thus, one is led to conclude that despite its advantages (independence from the virialization condition and from the hypothesis of sphericity) over other dynamical methods, such as the study of rotation curves of galaxies and of the motion of galaxies in groups, the weak gravitational lensing method (see eg. [6] for a critical assessment) has not been able, so far, to provide an unequivocal evidence of the presence of dark matter.

In what follows we shall argue that these most recent results are consistent with what one should expect from models where the presence of dark matter is replaced, at least partially (see eg. [7]), by the dynamical effects associated with the scale-dependence of the gravitational coupling arising from various models of quantum gravity.

In many approaches to quantum gravity the gravitational coupling turns out to have a momentum or scale dependence. In 1-loop higher-derivative quantum gravity this dependence arises as the theory has logarithmic divergences and the corresponding β-functions are negative meaning that 1-loop quantum gravity models are asymptotically free [8]. Similar conclusions are drawn from applying exact renormalization group techniques to gravity [9, 10]. This implies that the coupling constants of higher-derivative theories of gravity are actually momentum or scale-dependent. For an appropriate choice of the "confining" scale, the running of the coupling constants and in particular of the gravitational coupling can manifest itself macroscopically, as suggested in Ref.[12]. This behaviour is consistent with the absence of screening in gravity. Moreover, this implies that all classical equations depending on G will also exhibit a dependence on scale.

The cosmological implications of this feature of quantum gravity models were discussed in Refs. [13, 14] and compared with phenomenology in [7, 15]. In particular, it was pointed out in the latter that the behaviour of the gravitational coupling with scale, as suggested in [12], is

consistent with known astrophysical and cosmological bounds but still requires the presence of dark matter in the halo of spiral galaxies to explain the flatness of their rotation velocity curves, although that can be ensured with about 45% less dark matter [7, 11].

Let us now turn to the discussion of the gravitational lensing phenomena. Gravitational lensing is based on the prediction of General Relativity that light passing at a distance b from a spherical mass distribution is deflected by an angle that, in lowest order, is given by:

$$\alpha = \frac{4GM(b)}{bc^2} \ , \tag{1}$$

where $M(b)$ denotes the mass contained in b. The mass, $M(r)$, as a function of the distance from the centre can be inferred from Kepler's Third Law, $M(r) = rv^2/G$, where v is the rotation velocity of a mass at an orbit of radius r (see Ref. [7] for a discussion in the context of scale-dependent gravitational coupling models). Hence, assuming the lenses are spiral galaxies whose rotation velocity is constant for $r > 10 - 20\ kpc$ and that $\rho(r) \propto r^{-2}$, then:

$$\alpha = 2\pi \left(\frac{v}{c}\right)^2 \ , \tag{2}$$

which is independent of the impact paramenter, b, and G (see for instance [16]). For typical values of the rotation velocity of spiral galaxies, $v \sim 250\ km\ s^{-1}$, one finds $\alpha \approx 1''$. Essentially similar results would follow if elliptical galaxies are considered as the lenses instead, provided the rate of change of their total energy is unimportant, such that one can still use the Virial Theorem to evaluate their mean square velocity (see eq. (7) below).

Other lensing phenomena, such as the Einstein Rings, Crosses and Arcs are all given in terms of α [17] and a ratio of geometrically relevant distances and are therefore G-independent too. It then follows that one should not expect, at least in lowest order, any scale dependence in the measured lensing parameters. The same can be said about the so-called shear, which for any of the models discussed in the literature (power law or de Vaucouleurs), is G independent [2]. Hence, once again, one should not expect any scale dependence on this quantity as well. Since most recent observations [4] indicate that the distortion of the surface brightness of background galaxies can be accounted for by internal dynamical effects other than dark matter, and as the running of G mimics the presence of dark matter, consistency with observations implies, as shown, that the scale dependence of G does not affect gravitational lensing phenomena. A scale dependence on the gravitational lensing phenomena can nevertheless be inferred, if in order to estimate the relevant distances involved, the Hubble constant is used (see discussion in Ref. [7]) as this quantity is scale dependent (cf. eq. (9) below).

Of course, one should expect to draw conclusions about the presence of dark matter and/or about a possible scale dependence of G from other dynamical studies, such as the ones based on

the Virial Theorem. Indeed, the so-called Cosmic Virial Theorem is based on the assumption that particles with mass m_i and position vectors $\vec{x}_i$ that are under the action of a gravitational potential between two particle densities $\rho(x_i)$ given by

$$W = -\frac{1}{2}\frac{G}{M}a^5(t)\int d^3x_1 d^3x_2 \frac{[(\rho(x_1) - \rho_B)(\rho(x_2) - \rho_B)]}{x_{12}} \ , \tag{3}$$

where $M = \sum_i mi$, $a(t)$ is the scale factor of the Universe and ρ_B an average background density, can be described by the Hamiltonian function

$$\mathcal{H} = M(K + W) \tag{4}$$

with

$$K \equiv \frac{1}{2}\frac{\sum m_i (a\dot{\vec{x}}_i)^2}{M} \ . \tag{5}$$

Furthermore, by conveniently writing W in terms of the well known mass auto-correlation function $\xi(x)$,

$$W = -\frac{1}{2}G\rho_B a^2 \int d^3x \frac{\xi(x)}{x} \ , \tag{6}$$

one can obtain the Layzer-Irvine equation for the variation of the total energy [18]:

$$\frac{d}{dt}(K + W) + H(2K + W) = 0 \ , \tag{7}$$

where $H(t) \equiv \dot{a}(t)/a(t)$ is the Hubble parameter.

Equation (7) allows one to infer the rate of change of the total energy in terms of the Hubble parameter and the quantity 2K+W. The standard Virial Theorem readily follows from (7) if one neglects the rate of change of the total energy. Moreover, this equation also allows one to obtain an expression for the mass-weighted mean square velocity [18]

$$\bar{v}^2 = 2\pi G\rho_B J_2(x) \ , \tag{8}$$

where $J_2(x) = \int_0^x dx x\xi(x)$.

From the above discussion on the scale dependence of G, one has that the Friedmann equation, which describes the evolution of expansion rate of the Universe in terms of the matter energy density in an homogeneous and isotropic space-time should also, due to the presence of the gravitational coupling, exhibit a scale dependence. Considering for simplicity a spatially flat

Universe, one has [2]:

$$H^2(l) = \frac{8\pi G(l)}{3}\rho_B \ . \tag{9}$$

It then follows that one can readily generalize eq. (7):

$$\frac{d}{dt}(K + W(l)) = -H(l)(2K + W(l)) \ . \tag{10}$$

This generalization of the Layzer-Irvine equation indicates that the variation of the total energy, $K + W(l)$, is enhanced by the scale dependence of the rate of expansion $H(l)$ and by the $G(l)$ dependence in W. In order to estimate the effect of this dependence, we use the following fit for $G(l)$ in terms of the proper distance, l [12] (see also Refs. [7, 15, 23]):

$$G(l) = G\left[1 + 0.3\left(\frac{l}{kpc}\right)^{0.15}\right] \ . \tag{11}$$

Thus, for a system like MS 1224.7+2007 which stretches over $l = 2Mpc$ [2] we get, $G(l = 2Mpc) = 1.94G$, meaning that standard virial analysis of the system, based on eq. (8), underestimates the mass-weighted mean square velocity by a factor 2.

In summary, we have seen that in models where the gravitational coupling is scale dependent, predictions about the gravitational lensing and shear are, at least to lowest order, essentially similar to those of General Relativity. This implies that the most recent negative results of the use of gravitational lensing methods to infer the presence of dark matter are consistent with our predictions. We believe however, that the situation is, at cosmological scales, significantly different as the effective gravitational coupling is in this case significantly greater than Newton's constant. That can be used to account for features of the large scale structure of the Universe without invoking large amounts of dark matter [13].

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[2] Notice that from this Friedmann equation it implies that the Hubble constant has itself a scale dependence. As far as observations are concerned, this possibility cannot yet be ruled out before the discrepancy betweeen results obtained using as standard candles Type I supernovae (from which follows that $H_0 = (57 \pm 4)$ km s^{-1}Mpc^{-1} [19]) and classical Cepheid variables (that yield $H_0 = (82 \pm 17)$ km s^{-1}Mpc^{-1} [20]) is fully understood. The recent discovery of a correlation between the peak luminosity and the luminosity decay of Type I supernovae does imply that values of the Hubble constant obtained using those stars as standard candles tend to become higher, $H_0 = (67 \pm 7)$ km s^{-1}Mpc^{-1} [21], opening up the possibility for a convergence between the two methods. This is the main thrust behind the Supernova Cosmology Project [22]. In case the Hubble constant is shown to have the same value at all scales, then one has either to give up the idea that G has a scale dependence or instead the simple block renomalization procedure used here to generalize the classical equations (or also to put the burden to explain the scale independence of H on ρ_B).

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